MICROELECTRONIC SYSTEMS DESIGN RESEARCH GROUP

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Hardware Accelerators for Financial Mathematics

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The Computational Problem in Financial Mathematics

Modern (electronic) financial markets are as vivid as never before

- Calculation of market prices for financial products like options/derivatives
- Solid mathematical background based on stochastic differential equations
- Asset prices change within several microseconds today
- Accurate and fast calculations are key

Deutsche Bank announcement March 2011

“Whereas most low latency trading solutions are built in software, Deutsche Bank’s solution is based upon a field programmable gate array (FPGA)….”

“…this is a bit of a revolution, since it’s breaking a barrier….we’re getting into the low-single digit microseconds. That has never done before”

Automated Trader Journal: “FPGA Acceleration of European Options Pricing”

But they also have an energy problem!

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The Energy Problem in Financial Mathematics

Canary Wharf: 250MW
90% growth in Energy over next 5-7 Years

(The Guardian.co.uk)

- Olympic games in London 2012 with estimated 400MW power consumptions
- Olympic games get priority for power infrastructure
- Financial companies get shortages in access to power grid
  ⇨ Non-essential computing functions such as backup are being moved away from trading center to save power

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Financial Mathematics and MPSoC

- High performance computing under power constraints
- Banks start to use hardware accelerators
  - GPUs are already in use
  - FPGAs are currently evolving (MATLAB/HDL generator)
- Stochastic systems are known in MPSoC
  - E.g. communication systems
- Design methods already adapted concepts from financial mathematics
  - Modeling of dependencies: copula concept, upper tail dependency
  - Bounds for distribution functions: Comonotonicity
- Advanced design methodologies for various implementation styles for HW accelerators: FPGA, ASIPs, IP….
- Metrics and benchmarks

Pricing for (European) Call Options with Barriers

Financial contract between buyer and seller

Buyer has option to buy e.g. a share from the seller at a certain time for a certain price (strike)

Arbitrage in finance: possibility of a risk-free profit at zero cost

Risk neutral price calculation for derivatives: no arbitrage possibility

Source: publik16 / www.flickr.com

Black-Scholes Model (Nobel Prize 1997)

Most option price calculation are based on Black Scholes Model

- Stochastic Differential Equations based on Brownian Motion
- Asset/share price not differentiable ⇨ integral formulation

$$dS_t = \mu S_t d_t + V_t dW_t$$

S_t: price of the asset/share

μ: expected rate of the return of the asset

V_t: volatility

W_t: Brownian motion

….in many cases lack of analytical solutions

⇨Monte Carlo simulations

- precision ~ #paths², precision $\varepsilon = \frac{1}{N}$ ⇨ $O(N^3)$
- *x* millions of path simulations necessary

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Heston's Model of Stochastic Volatility

Current GPU/FPGA accelerators use Black-Scholes Model

- Volatility in BS model is constant over time
- Crash situations can not be modeled

⇨ Heston model is today state-of-the-art in finance (1993)

⇨ Volatility modeled as stochastic differential equation

$$dS_t = \mu S_t d_t + \sqrt{V_t} S_t dW_t^a$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_t^b$$

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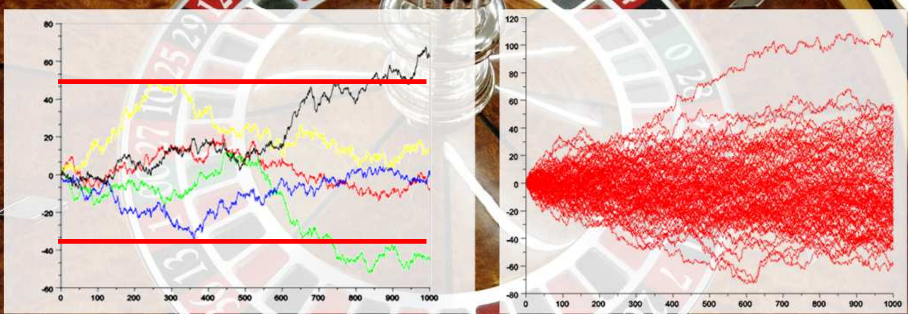
Brownian Motion (Particle Theory)

- Time continuous stochastic process with continuous paths
- Not differentiable with infinite length
- W(t) is normally distributed with N(0,t) i.e. integrate over Gaussian noise



"Efficient" random generators are key!

New Non-Uniform Random Number Generator

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Mersenne Twister, Box-Muller: high complexity

	Virtex-5 FX70T-3	**Virtex-4 LX100-12**			
	New architecture			Cheung et al. 2007	
	converter + ICDF lookup	converter	ICDF lookup	address decoding	ICDF lookup
Slices	44	55	[illegible]	[illegible]	[illegible]
Flip-flops	62	62	[illegible]	[illegible]	[illegible]
LUTs	40	55	[illegible]	[illegible]	[illegible]
DSP	1 (DSP48E)	0	2 (DSP48)	0	2 (DSP48)
BRAM	1 (36kb)	0	2 (18kb)	0	2 (18kb)
F_{max} / MHz	381	> 400	< 375	< 375	< 375

180 instead of 346 slices
⇨ 48% less area @ higher precision
@ proven quality

Parameters optimized for Xilinx Virtex-5 DSP48E

What about quality?

Importance of Quality Tests

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Uniformly distributed random numbers

Standard tests

Non-uniform random numbers

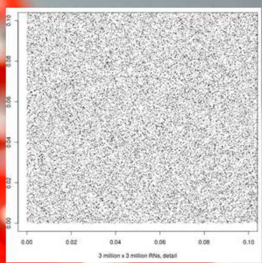
No standard tests!

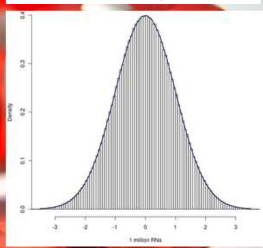
Various χ^2 (goodness-of-fit) tests

- Bin generated values into groups
- Compare with theoretical values

Kolmogorov-Smirnov test

- Compares generated values with theoretical cumulative distribution function





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Normal Random Number Generation

Implementation	Architecture	Power	Throughput [Msamples / s]	Energy per sample
Fast-Mersenne Twister optimized for SIMD Box-Muller	Intel Core 2 Duo PC 2.0 GHz, 3 GB RAM one core only	~ 100 W	**600**	**166.67 pJ**
Nvidia Mersenne Twister + Box-Muller CUDA	Nvidia GeForce 9800 GT	~ 105 W	**1510**	**69.54 pJ**
Nvidia Mersenne Twister + Box-Muller OpenCL			**1463**	**71.77 pJ**
Inversion method for general non-uniform distributions, only one instance	Xilinx FPGA Virtex-5FX70T3 @380 MHz	~ 1.3 W	**379**	**3.43 pJ**
Inversion method for general non-uniform distributions, 100 instances		~ 1.8 W	**37900**	**0.05 pJ**

Source: ~H. Crimson / www.flickr.com

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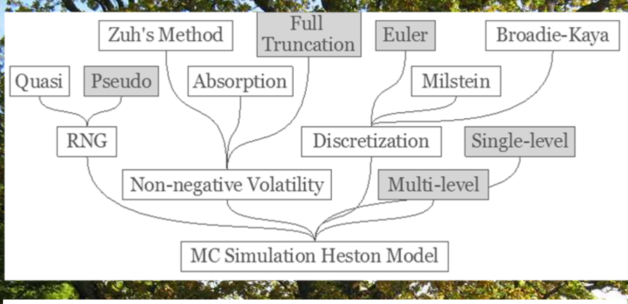
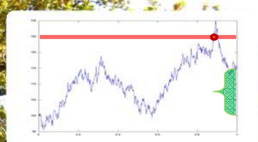
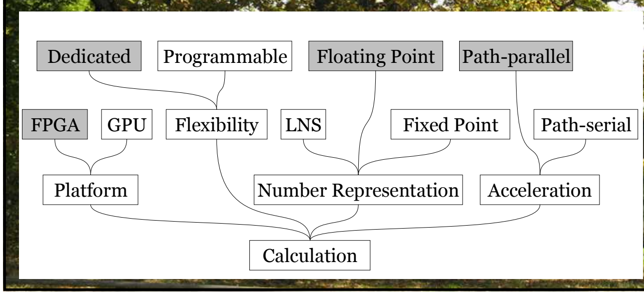
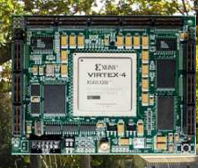
Design Space for Heston's Model









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Benchmarks

Fair comparison

- Benchmarks and metrics
- Not known in financial hardware accelerator community

Set up a benchmark set for Heston's model in cooperation with financial mathematician

- Comprehensive pricing tasks with real world scenarios
- In total 12 different call options: European call, single/double barrier...

Download on **www.uni-kl.de/benchmarking**
(includes Octave model)

Source: Mike Johnson - TheBusyBrain.com

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First Results

Multi-level Monte Carlo accelerator of Heston model (**full pricing**)

Precision $\varepsilon = \frac{1}{N} \Rightarrow MC\ O(N^3), Multilevel\ MC\ up\ to\ O(N^2)$

	Nvidia GPU Tesla 2050 (Opt. Open-CL)	Intel Xeon CPU 1 core@3.07 GHz (Matlab)
Real-time / s	0.332	22.41
Energy per Simulation / J	96.28	3742.5
Number Format	Floating Point Double Precision	Floating Point Double Precision

Multi-level simulation up to level 4 with antithetic variance reduction, continuity correction, precision of 0.01 and a total of 4,731,944 paths

Best of our knowledge first Multi-level MC implementation of Heston Model on CPU and GPU

FPGA Acceleration: Heston Model

Multi-level Monte Carlo Heston model (**only path acceleration**)

	Xilinx Virtex-5* FPGA@100MHz	Nvidia GPU Tesla 2050	Intel Xeon CPU 1 core@3.07 GHz
Real-time / s	**15.37**	**22.269**	1600
Energy per Path / mJ	**0.023**	**1.076**	44.55
Number Format	Fixed Point 32 bits	Floating Point Double Precision	Floating Point Double Precision

Fixed to a level 5 simulation (1024 fine and 256 coarse steps in parallel) with antithetic variance reduction on 6,000,640 paths

***preliminary results**, FPGA capacity allows 5 instances

Best of our knowledge first Heston model implementation on FPGA

Fixed Point versus Float

Multi-level Monte Carlo Heston model (**only path simulation**)

	Fixed Point* 32 bits		Floating Point* Single Precision	
	Number	**Percentage**	**Number**	**Percentage**
LUTs	6645	14%	10291	22%
Flip-Flops	5783	12%	12075	26%
LUT-FF pairs	8729	19%	13397	29%
DSP48E	25	19%	4	3%
BRAM36	7	4%	1	1%
MHz	96		104	

* **preliminary results** Virtex 5 – XC5VFX70T (on ML-507 evaluation board)

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Summary

Financial Mathematics exciting application for FPGA accelerators
Can profit from MPSoC community
Requires interdisciplinary teams

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http://ems.eit.uni-kl.de